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EMERGENCY LANDINGS FROM LOW ALTITUDES--MINIMUM ALTITUDE REQUIRED TO TURN BACK INTO FIELD IN CASE OF ENGINE FAILURE AFTER TAKE-OFF

(FLYING SECTION REPORT NO. 83)

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EMERGENCY LANDINGS FROM LOW ALTITUDES—MINIMUM ALTITUDE REQUIRED TO TURN BACK INTO FIELD IN CASE OF ENGINE FAILURE AFTER TAKE-OFF.

OBJECT OF INVESTIGATION.

An examination of records of accidents discloses the fact that a large percentage is due to the efforts of the pilot to turn back into the field when his engine fails on the take-off, without sufficient altitude to complete the turn. For each airplane there is a minimum altitude below which a complete 180° turn can not be made. The object of this investigation is to determine this altitude for various types.

SUMMARY OF RESULTS.

TABLE 1.—*Minimum altitudes for various types.*

Type.	Total weight.[1]	Minimum altitude.	Most efficient air speed.	Best angle of bank.	Radius of turn.
	Pounds.	*Feet.*	*M. P. H.*		*Feet.*
DH-4	4,297	340	75	45°	380
SE-5	2,058	270	70	45°	330
Curtiss JN4-H	2,200	230	60	45°	240
Thomas-Morse MB-3	2,548	400	78	45°	400
XB1-A	3,679	300	73	45°	360

[1] Full military load used in each case. If airplane is flown without full load, the altitude loss will be proportionately less.

The altitude given for each type should be taken as an absolute minimum for a complete turn of 180°, and can only be obtained by following fairly closely the air speeds and angles of bank which are recommended. Both theory and experiment point to the fact that a reasonable deviation from these conditions does not greatly increase the loss in altitude, and, with average piloting, an airplane can be turned back with safety at the altitudes shown in the table. On the other hand, even with exceptional piloting, these altitudes can not be appreciably decreased if a complete 180° turn must be made.

There is only one part of the maneuver in which a gain can be made, namely, the take-off itself. The pilot should so "play his field" on the take-off that a complete half turn will not be necessary.

DISCUSSION OF METHODS USED.

In the solution of this problem, recourse was had to both theory and experiment. Briefly, the method of attack was as follows: The minimum altitude lost in the turn, and the best combination of air speed, angle of bank, and radius of turn to give this minimum were computed from the airplane coefficients. Then the altitude lost was measured in actual flight.

Table 1 gives only altitudes which have been checked in flight. The agreement between the measured and computed values, however, is so close that estimates may be made for other types by means of the chart on the last page of this report. There was a discrepancy in the case of the Thomas-Morse, the computed value being almost 19 per cent lower than the measured value. In all other cases the check was well within the limit of error in instrument readings. A more complete discussion of the method of computation is given below.

MATHEMATICAL APPENDIX.

SYMBOLS USED.

H = Altitude lost in 180° turn.

V = Forward velocity of airplane in feet per second.

V_d = Vertical component of forward velocity in feet per second.

V_h = Horizontal component of forward velocity in feet per second.

θ = Angle of flight path with horizontal.

A = Area of wings in square feet.

d = Density of air in pounds per cubic foot.

K_y = Lift coefficient of whole airplane (absolute).

K_x = Drag coefficient of whole airplane (absolute).

r = Radius of turn in feet.

L = Total lift in pounds.

D = Total drag in pounds.

W = Total weight of airplane in pounds.

ANALYTICAL SOLUTION.

Figure 1 shows the forces acting on an airplane in a gliding turn. The drag D is balanced by the component of the weight in the direction of the flight path $W \sin \theta$. The resultant, R, of the lift, L, and the component of the weight perpendicular to the flight path, may be considered as an unbalanced force producing an acceleration toward the center of the turn.

The following relations are apparent from the figure:

(1) $\sin \Theta = V_d/V$

(2) $D = W \sin \Theta$

(3) $L^2 - W^2 \cos^2\Theta = R^2$

(4) $W \sin \Theta = K_x \frac{d}{g} A V^2$

(5) $W^2 \cos^2 \Theta + \frac{W^2}{g^2} \frac{V^4}{r^2} = Ky^2 \frac{d^2}{g^2} A^2 V^4$

(6) $V_d = \frac{K_x\, d\, A\, V^3}{g}$

Combining (4) and (5), and dividing by V^4, gives

(7) $$\frac{K_x^2\, d^2\, A^2}{g^2 \tan^2 \Theta} + \frac{W^2}{g^2\, r^2} = Ky^2 \frac{d^2}{g^2} A^2$$

now

$$H = \pi\, r \tan \theta.$$

or

(8) $$\tan^2 \theta = \frac{H^2}{\pi^2\, r^2}$$

For simplification, the following notation is introduced:

$$a = (d\, A\, \pi)^2$$
$$b = (d\, A)^2$$
$$c = W^2$$

Equation (10) becomes

(11) $$H^2 = \frac{a\, K_x^2\, r^4}{b\, K_y^2\, r^2 - c}$$

differentiating

(12) $$\frac{\delta H^2}{\delta r} = \frac{4\, a(br^2ky^2 - c)K_x^2r^3 - 2\, abr^5kx^2ky^2}{(br^2ky^2 - c)^2} = 0$$

(13) $$\frac{\delta H^2}{\delta K_x} = \frac{2\, a(br^2ky^2 - c)r^4kx - 2\, abr^6kx^2kyky'}{(br^2ky^2 - c)^2} = 0$$

Combining (12) and (13), and simplifying

(14) $$r^3(ky^2 - k_xkyky') - r^2kx\, ky^2 - r\frac{c}{b} - 2\,\frac{c}{b}kx = 0.$$

Neglecting $r^2\, kx\, ky^2$ and $2\,\frac{c}{b}\, kx$, which are relatively small, and solving for r, we have

(15) $$r = \left(\frac{c}{b(ky^2 - kx\, ky\, ky')}\right)^{\frac{1}{2}} = \frac{W}{dA\,(K_y^2 - kx\, ky\, ky')^{\frac{1}{2}}}$$

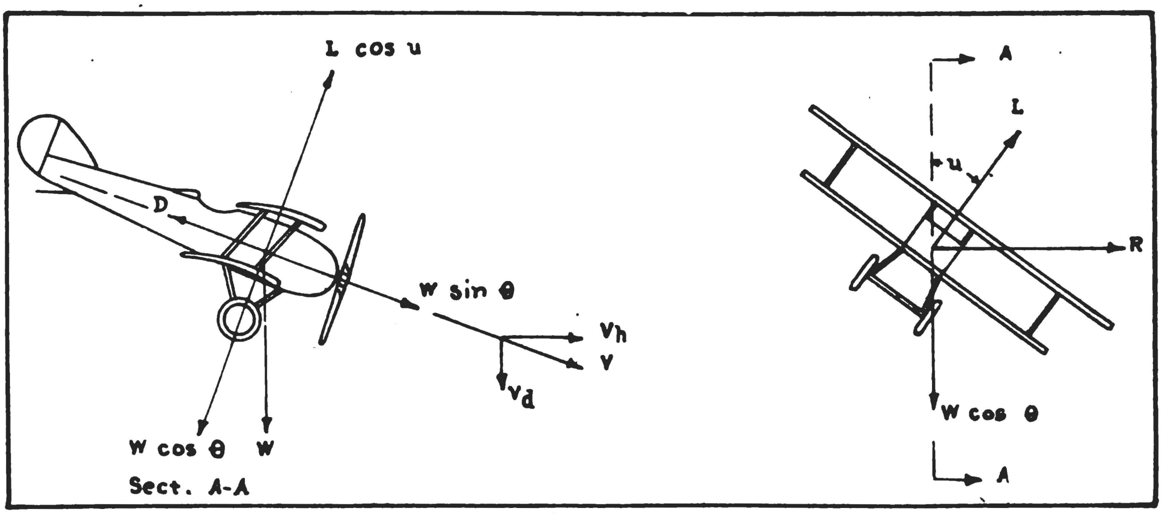

Fig. 1.

substituting in (7)

(9) $$\frac{K_x^2\, d^2\, A^2\, \pi^2\, r^2}{g^2\, H^2} = K_y^2 \frac{d^2}{g^2} A^2 - \frac{W^2}{g^2\, r^2}$$

or

(10) $$H^2 = \frac{K_x^2\, d^2\, A^2\, \pi^2\, r^4}{K_y^2\, d^2\, A^2\, r^2 - W^2}$$

For a given airplane, flying in a given density, d, A and W are constant. The variables entering into equation (10) are therefore r, K_x and K_y. Of these three, r and K_x may be taken as independent variables, K_y being considered as a function of K_x. In order that H may be a minimum it is only necessary that

$$\frac{\delta H}{\delta r} = 0$$

and

$$\frac{\delta H}{\delta K_x} = 0$$

A solution of these two equations simultaneously will therefore give the radius of turn at which the least altitude will be lost as well as the best angle of incidence at which to fly during the turn.

Substituting in (10), and simplifying

(16) $$H^2 = \frac{\pi^2 W^2}{d^2 A^2} \cdot \frac{K_x^2}{(kx\, ky^3\, ky' - kx^2ky^2ky'^2)}$$

This equation is of interest, as it brings out the fact that for a given combination of airplane coefficients, the altitude lost in a 180° turn is directly proportional to the wing loading and inversely proportional to the density.

Differentiating with respect to K_x, equating to 0, and simplifying, we have

(17) $$K_y^3ky' - 3\, kx\, ky^2ky'^2 + 2\, kx^2ky\, ky'^3 - kx\, ky^3ky'' + 2\, kx^2ky^2ky'ky'' = 0.$$

We will now assume that ky can be expressed as a function of kx, of the form

(18) $$K_y = a + b\, kx + c\, kx^2$$

While such an expression will not hold true over the whole range of values of K_x, it will approximate the curve very closely over the range where the minimum value of H occurs.

Differentiating (18)

(19) $$ky' = b + 2\, c\, K_x$$

(20) $$K_y'' = 2\, c.$$

Substituting (18), (19), and (20) in (17) and collecting terms.

$$(21)\quad 12\,c^4kx^7+25\,bc^3kx^6+(16\,b^2c^2+8\,ac^3)\,kx^5+(3\,b^3c+3\,abc^2)\,kx^4-(4\,ab^2c+4\,a^2c^2)kx^3-(ab^3+5a^2bc)kx^2+a^3b=0$$

In this equation the constants a, b, and c can be found for any airplane whose lift-drag curve is known, and the equation can be solved for kx. This value of kx can then be substituted in equation (16) and the minimum value of H can be found.

For example, in the case of the DH-4

$$a=-.186$$
$$b=15.27$$
$$c=-84.4$$

Substituting these values in equation (21) and solving, we find

$$Kx=.07$$

which can be substituted in equation (16), giving a value of H=260 feet.

A correction must now be made for the conditions at the beginning and end of the maneuver.

The recovery.—It has been found that the excess speed in the turn, which is usually from 15 to 20 miles per hour, can be used in coming out of the bank without loss in altitude. It is only necessary, then, to allow an altitude above the ground equal to one-half the span of the airplane in order to allow the lower wing to clear the ground. The average correction will be about 12 per cent of the total altitude.

A further arbitrary addition of 10 per cent will be added as a safety factor to allow for inaccuracies in piloting, making a total correction of 32 per cent.

With this correction the value of H for the DH-4 is 343 feet.

At first sight this method of correcting for conditions at the beginning and end of the maneuver may seem very approximate, but allowing an error as large as 25 per cent in each of the corrections, the probable error in the total altitude from this cause will be less than 5 per cent, which means an average error in altitude of only 15 feet. The final test of any assumption is the accuracy of the results which it gives, and except in the case of the Thomas Morse, as noted above, the agreement with the results of actual tests has been very close.

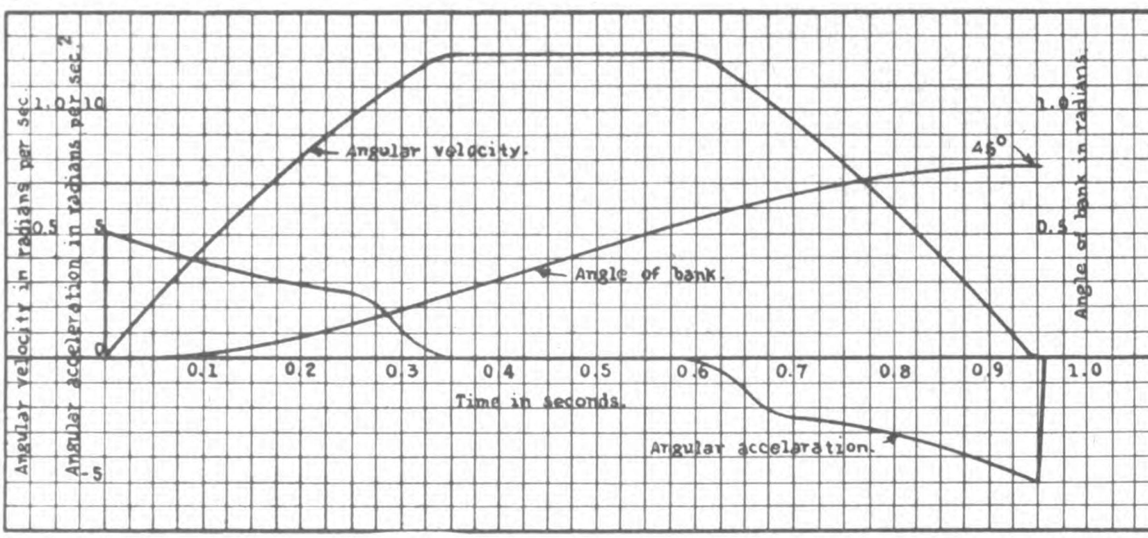

FIG. 2. Lateral control, XB1-A. Showing angular velocities and accelerations during a bank of 45 degrees.

The start. Immediately after the failure of the engine there is an interval during which the pilot must nose over to prevent stalling. During this time there is always a drop in air speed, which varies with the alertness of the pilot and the characteristics of the airplane. Tests were made to determine this drop in air speed under conditions simulating sudden engine failure, and the average drop for the DH-4 and the XB1-A was 5 miles per hour. The altitude lost in regaining this speed was computed as 14 feet for the DH-4, or 5 per cent of the total altitude.

In banking, there is a further loss of altitude. The time of bank from 0 to 45° was computed for the XB1-A by the method of step-by-step integration, using a computed lateral radius of gyration of 5.5 feet. The curves of angular velocity and angular acceleration are shown in Fig. 2. The computation gave 0.96 second to 45°, while a rough measurement in flight gave 1.1 seconds. Half of this time may be considered as not producing any turning. The total time for the turn being 11 seconds, the loss is 5 per cent of the total.

The above method is perfectly general, and can be applied to any airplane whose weight, wing area, and lift-drag curve are known. It is, however, very laborious, as it involves the solution of a seventh-degree equation. The following graphical method, while not as general, is much more easily applied and can be used for types equipped with R. A. F. 15 or sections of approximately similar characteristics.

GRAPHICAL SOLUTION.

Equation (18), between lift and drag coefficients will be used as the basis of the graphical solution:

$$(18)\qquad ky=a+b\,kx+c\,kx^2$$

For the DH-4

$$(22)\qquad ky=-.186+15.27\,kx-84.4\,kx^2$$

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A plot of this equation is shown in Fig. 3. It will be noted that the agreement with the experimental results is very good, except in the region of maximum ky. At $kx=.07$, where the minimum value of H occurs, the empirical curve agrees in slope and location with the actual curve.

For any other given type equipped with R. A. F. 15 section it is assumed that the difference between its drag and the drag of the DH-4 is a constant for all values of ky, that is,

$$(23) \qquad kx_1=kx+\mathrm{K}$$

(Subscripts refer to any given type other than DH-4.)

The value of K depends upon the parasite areas of the two types.

$$(29) \quad \mathrm{K}=0.64\left[\left(\frac{Ae}{A}\right)_1-\left(\frac{Ae}{A}\right)\right]$$

Ae=equivalent flat plate area of parasite resistance.

For the DH-4,

$Ae=14$

$A=440$

$\frac{Ae}{A}=.0318$

Information is available on the equivalent parasite area of most types which have been flight-tested at McCook Field. It is more convenient, however, to use fineness

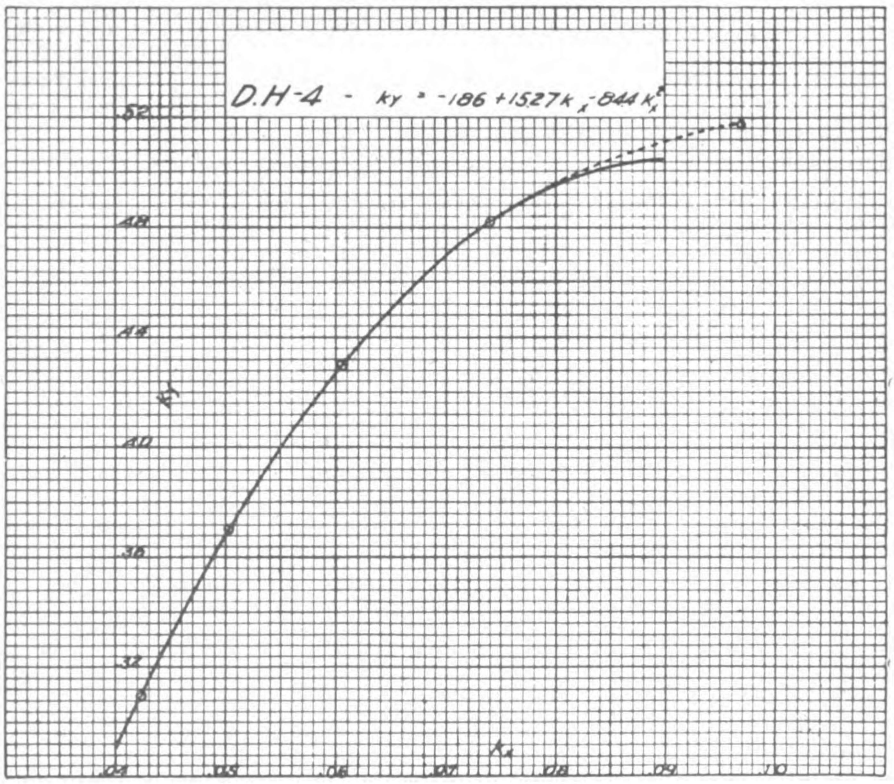

FIG. 3.

Equation (18) will be of the form:

$$(24) \qquad ky=a_1+b_1 kx_1+c_1 kx_1^2$$

Substituting (23) in (24)

$$(25) \quad ky=a_1+b_1 kx+b_1\mathrm{K}+c_1 kx^2+2c_1\mathrm{K}kx+c_1\mathrm{K}^2$$

Combining (18) and (25), and equating coefficients of like powers of kx,

$$(26) \quad a_1=a-b\mathrm{K}+c\mathrm{K}^2$$
$$(27) \quad b_1=b-2c\mathrm{K}$$
$$(28) \quad c_1=c$$

as a parameter, fineness being a function of $\frac{Ae}{A}$. The following table gives the values of $\frac{Ae}{A}$, K, and the coefficients of equation (24), for various values of fineness:

TABLE 2.

Fineness.	$\frac{Ae}{A}$	K	a_1	b_1	c_1
90	.050	.01164	−.3754	17.23	−84.4
100	.0326	.000512	−.1938	15.36	−84.4
110	.0225	−.00595	−.0980	14.27	−84.2
120	.0153	−.01055	−.0343	13.49	−84.1

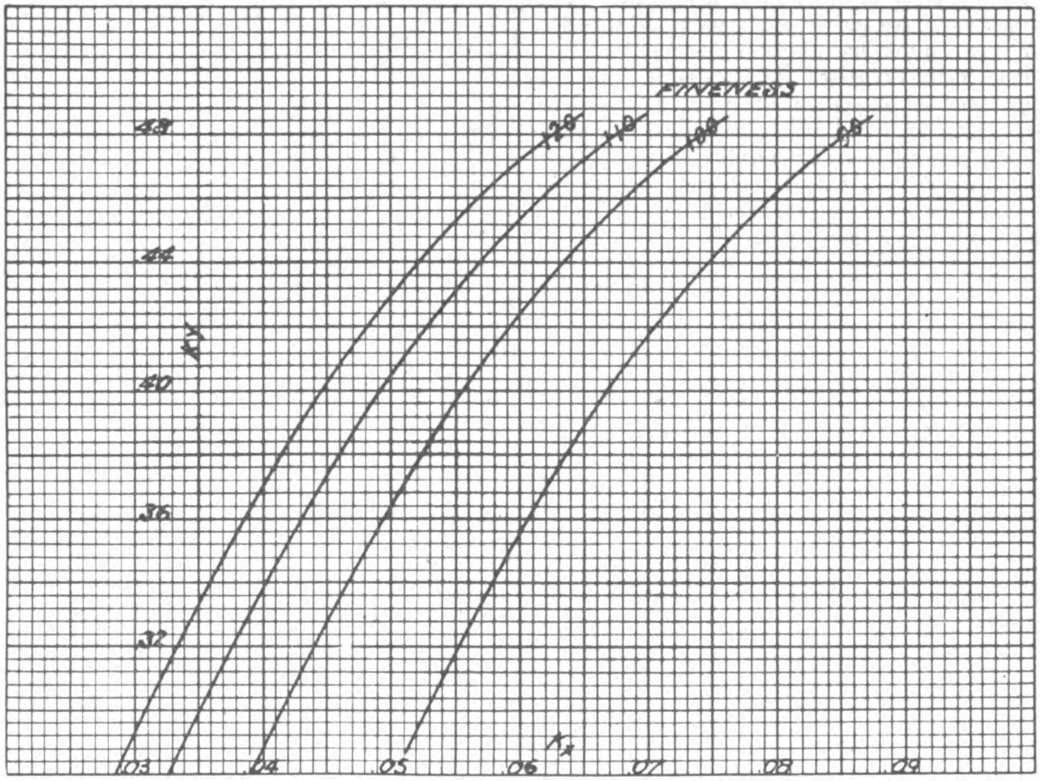
FINENESS
120
110
100
98
48
44
40
36
32
Ky
Kx
.03
.04
.05
.06
.07
.08
.09

FIG. 4.

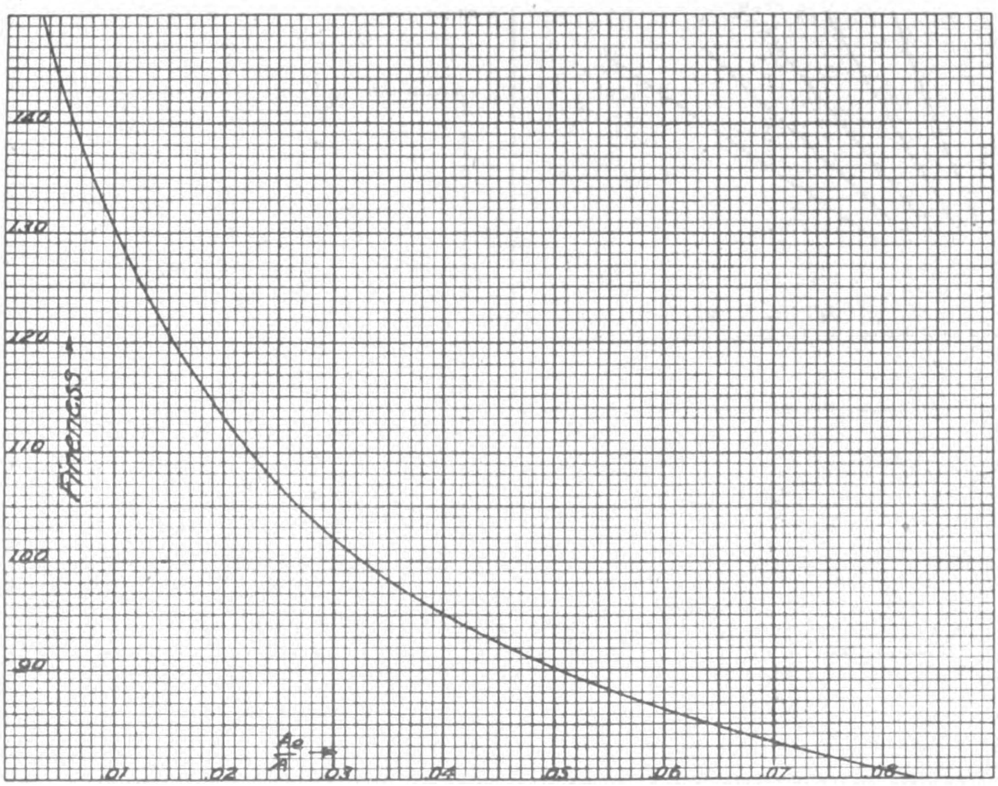
140
130
120
110
100
90
Fineness
.01
.02
.03
.04
.05
.06
.07
.08

FIG. 5.

Fig. 4 shows polar diagrams for various finenesses. It is realized that these curves are approximations, since they are based on the assumption of equation (23).

This same method of attack has, however, been used in the Kerber method of performance prediction,[1] and has given excellent results. A report containing a complete discussion of the relation between fineness and parasite area, and of the method of determination of fineness with any air foil is now being prepared by Mr. Kerber, and will be published within a short time.

The relation between fineness and $\frac{Ae}{A}$ is shown in Fig. 5.

From equation (16),

$$H=\frac{W}{d\,A}\,\frac{K_x}{K_y(kx\,ky\,ky'-K_x^2K_y'^2)^{\frac{1}{2}}}$$

Assuming standard density of 0.0761 pounds per square foot,

$$(30)\qquad H=41.2\frac{W}{A}f$$

where

$$f=\frac{K_x}{K_y\,(kx\,ky\,ky'-K_x^2k_y'^2)^{\frac{1}{2}}}$$

Equation (30) may be plotted with H and f as coordinates and $\frac{W}{A}$ as a parameter, giving a series of radiating straight lines, as shown symbolically in Figure 6. Section (2) of Figure 10 is plotted in this manner.

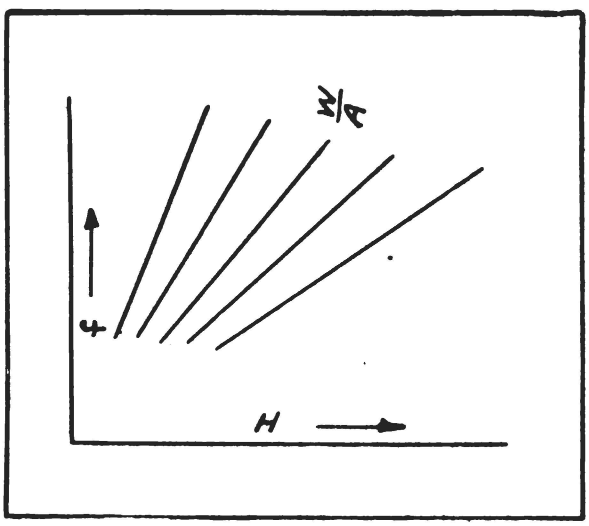

FIG. 6.

Rewriting equation (15),

$$r=\frac{W}{d\,A(K_y^2-Kx\,ky\,ky')^{\frac{1}{2}}}$$

or

$$(31)\qquad r=13.1\frac{W}{A\,f'}$$

where

$$f'=(K_y^2-kx\,ky\,ky')$$

Combining equations (30), (31), and (8),

$$(32)\qquad H=r\,f\,f'$$

which may be plotted as shown in Figure 7.

(See section (4) of chart on page 10.)

[1] See Air Service Information Circular, Vol. II, No. 183.

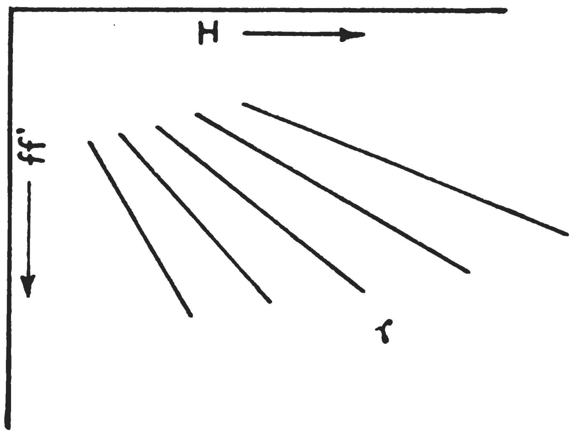

FIG. 7.

Rewriting equation (4),

$$V=\frac{W^{\frac{1}{2}}}{A}\left(\frac{\sin\theta\,g}{K_x\,d}\right)^{\frac{1}{2}}$$

or

$$(33)\qquad V=\frac{W^{\frac{1}{2}}}{A}f''$$

where

$$f''=\left(\frac{\sin\theta\,g}{K_x\,d}\right)^{\frac{1}{2}}$$

which may be plotted as shown in Figure 8.

(See section 6 of fig. 10.)

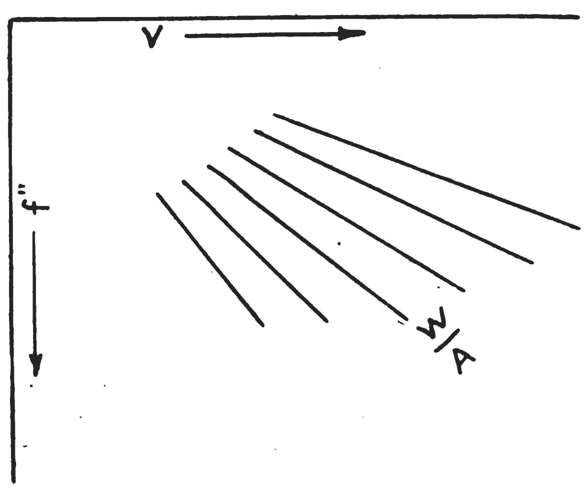

FIG. 8.

Let u—angle of bank

$$(34)\qquad \tan u=\frac{V^2}{g\,r}$$

(Actually, V_h^2 should be used, but the slight increase in accuracy does not justify the added complication.)

Substituting (31) and (33) in (34),

$$(35)\qquad \tan u=.00237\,f'f''^2.$$

The quantities f, f', and f'' are functions of the lift and drag coefficients. In Table 2, fineness has also been shown to be a function of these coefficients. That is:

$$f''=f_1\,(K_x,\ K_y)$$
$$f'f''^2=f_2\,(K_x,\ K_y)$$
$$F=f_3\,(K_x,\ K_y)$$

We have here three equations in five variables. An elimination of any two would give one equation in three variables. For example, an elimination of K_x and K_y would give a relation between F, $f'f''^2$ and f'', which could be plotted as shown in Figure 9.

Section 5 of Figure 10 is obtained in this manner, except that the abscissae are angles of bank instead of values of the function $f'\,f''^2$. This same line of reasoning applies to sections (1) and (3).

It will be noted that plotting by this method would be an extremely complicated process, owing to fact that no simple analytical expression can be found for the relationship of Figure 9. In the actual construction of the chart, the computations were greatly simplified by reversing the process. For each fineness, values of K_y were computed for various values of K_x, by use of equation (24) and Table 2. The functions f, f', etc., were then computed for these values of the coefficients, and corresponding values of H, r, and V were found.

In using the chart, it is only necessary to know the wing loading and the fineness. If the fineness is not known, it can be found from the high speed of the airplane, as outlined in Air Service Information Circular, Vol. II, No. 183,

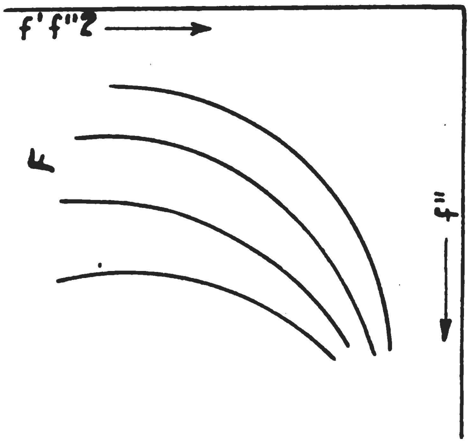

FIG. 9.

or, if the parasite resistance is known, fineness can be found by use of Figure 5. Knowing the fineness and wing loading, and assuming an angle of bank (45 degrees will give best results), H, r, and V can be found as shown by the dotted example line.

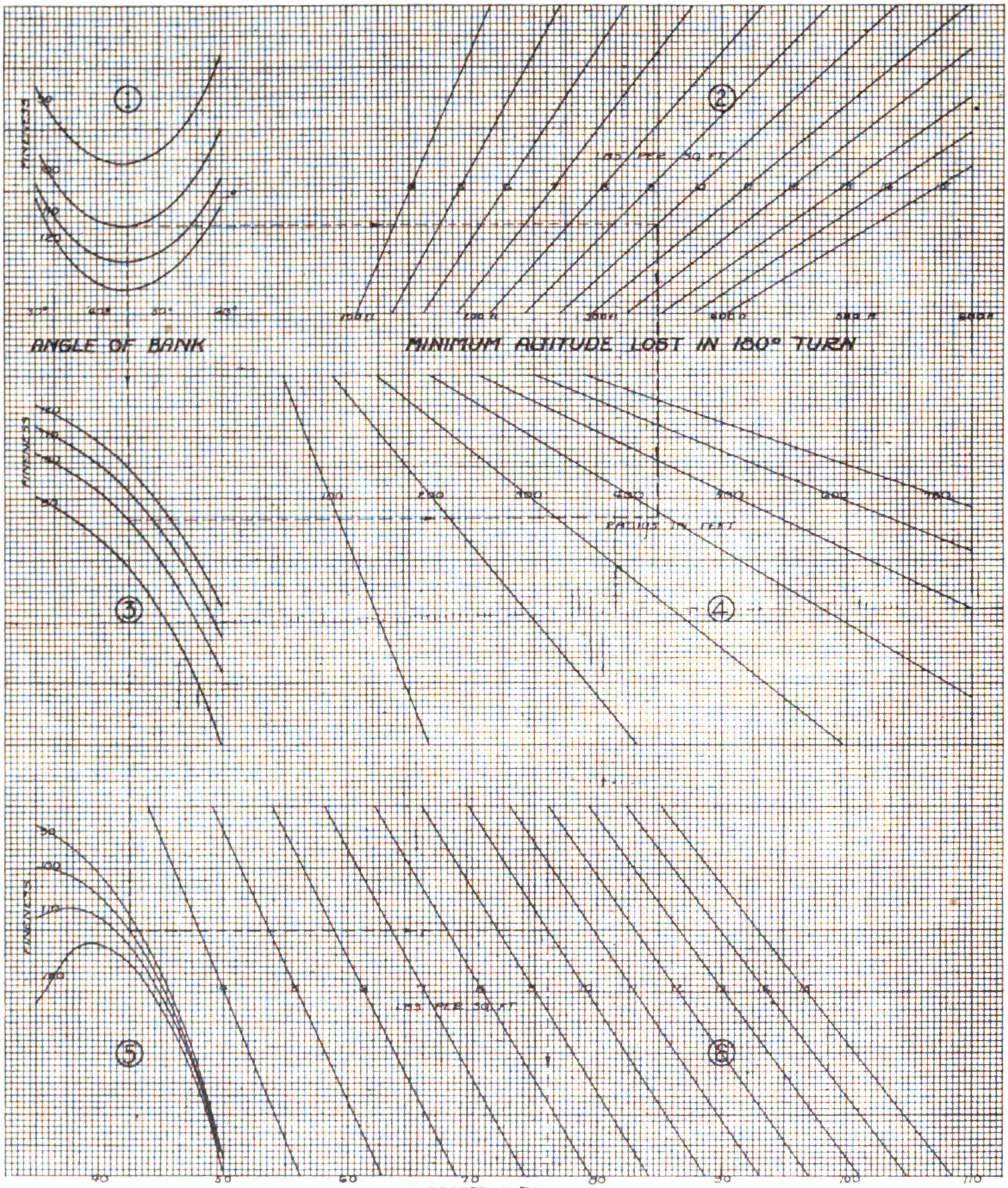

FIG. 10.—Chart for prediction of minimum altitude lost in 180° turn.